LESSON 2-1
Vertex Form of a Quadratic Function

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Program Resources

Lesson Overview

KEY CONCEPTS

Mathematics Objectives

- ✔ Create quadratic functions in vertex form to represent relationships between variables as shown in their graphs.
- ✔ Graph functions on coordinate axes using their key features.
- ✔ Interpret key features of the graph of a quadratic function.

Language Objective

- ✔ Listen to instruction and discussion with peers with a goal of identifying connections between the vertex form of a quadratic function and the graph of the function.

Essential Understanding

All quadratic functions are transformations of the parent function $f(x) = x^2$. The vertex form of a quadratic function shows how the graph of the parent function can be transformed.

COHERENCE

Previously in earlier courses, students:

- Created and graphed linear functions.

In this lesson, students:

- Create and graph quadratic functions in vertex form by identifying and interpreting the key features.

Later in this topic, students will:

- Rewrite quadratic functions in vertex form using the standard form.

BALANCE

This lesson emphasizes a blend of *conceptual understanding* and *procedural skill.*

- Students understand that all quadratic functions are transformations of the parent function.
- Students interpret key features of the graph of a quadratic function in terms of the quantities given in real-world problems, such as those describing projectile motion.

Presentation is available at SavvasRealize.com.

Math Talk

For use before this lesson.

Find a Rule Lesson 2-1

What rule describes the number pairs in each table?

x	0	1	2	3	4
y	8	2	0	2	8

What is the same?

What is different?

x	0	1	2	3	4
y	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2

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TEKS 2A.4B

Find a Rule As students work on each rule, look for students who identify that both tables contain the number pair (2,0). Look for students who correctly write the quadratic equations $y = 2(x - 2)^2$, $y = \frac{1}{2}(x - 2)^2$. Ask students to identify similarities and differences in the tables and in the equations.

Skills Review & Practice

Present or assign to reactivate prior knowledge.

- **F08** Point-Slope Form and Writing Linear Equations

Vocabulary Builder

Use the Vocabulary Builder activity online to support understanding of the terms *parabola*, *quadratic function*, *vertex form of a quadratic function* and the review term *vertex*.

Texas Essential Knowledge and Skills (TEKS)

Quadratic and square root functions, equations, and inequalities. The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

2A.4D transform a quadratic function $f(x) = ax^2 + bx + c$ to the form $f(x) = a(x - h)^2 + k$ to identify the different attributes of $f(x)$.
Also **2A.4B** and **2A.7I**

Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

2A.1A apply mathematics to problems arising in everyday life, society, and the workplace.
2A.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

English Language Proficiency Standards (ELPS)

WRITING 4E Write content-area specific text using conventions such as capitalization, punctuation, and abbreviations and grammatical structures such as subject-verb agreement, verb tense, possessive case, and contractions.

College and Career Readiness Standards

VI.B.2 Algebraically construct and analyze new functions.
Also **VI.A.2, VI.B.1, II.D.2**, and **VI.C.2**

Standards in **VII** Problem Solving and Reasoning, **VIII** Communication and Representation, and **IX** Connections are infused throughout the program.

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Activity

EXPLORE & SHARE

INSTRUCTIONAL FOCUS Students use their knowledge of the parent function $f(x) = x^2$ to explore how adding or multiplying the function by a constant changes the value of the function.

STUDENT COMPANION Students can do their work for the task on their *Student Companion* pages, which you can print from **SavvasRealize.com**.

Before WHOLE CLASS

Implement Tasks That Promote Reasoning and Problem Solving

Q: What does the table show?
[Each column shows an input-output pair for the function and a picture showing how they are connected.]

Q: Do you see any patterns in the table?
[The value of $A(x)$ in the third row is equal to the square of the value of x in the first row.]

During SMALL GROUP

Support Productive Struggle in Learning Mathematics

Q: Why is it helpful to think of the new function described in Part *A* in terms of $A(x)$?
[Sample: You know that $A(x) = x^2$. The value of the new function is $2x$ times x or $2x^2$, so $f(x) = 2A(x)$. Therefore, if you know $A(x)$, you can find $f(x)$ by multiplying $A(x)$ by 2.]

Q: What does $A(x) + 2$ mean in relation to $A(x)$?
[Find the value of $A(x)$, then add 2 to that value.]

For Early Finishers

Q: Suppose you double both the length and the width of the square to make a larger square. How are the corresponding areas affected? [The corresponding areas are 4 times as great.]

After WHOLE CLASS

Facilitate Meaningful Mathematical Discourse

Facilitate a discussion with students about their answers to Parts B and C.

Q: What strategy can you use to determine the relationship between the new function described in Part A and $A(x)$?
[Add another row to the table for the values of $f(x)$. Each value of $f(x)$ is twice the value of $A(x)$. So $f(x) = 2A(x)$.]

Q: What strategy can you use to determine the value of $f(x) + 1$ if you know the value of $A(x)$?
[Multiply the value of $A(x)$ by 2 and then add 1.]

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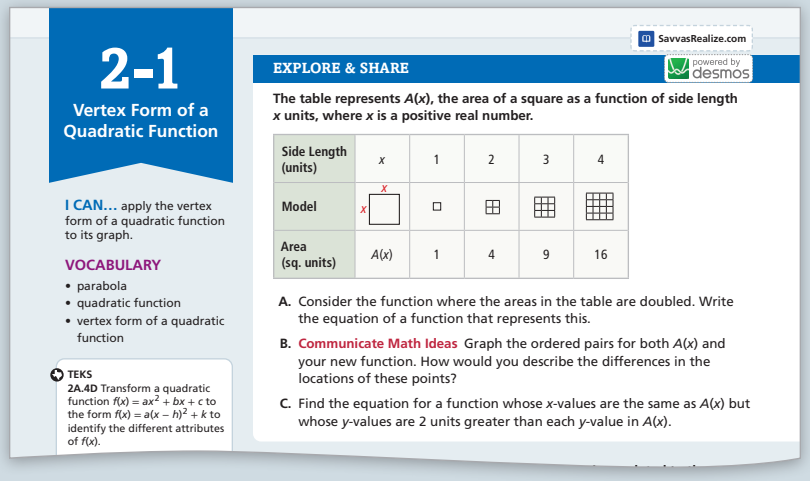
STUDENT EDITION



2-1 Vertex Form of a Quadratic Function

I CAN... apply the vertex form of a quadratic function to its graph.

VOCABULARY
- parabola
- quadratic function
- vertex form of a quadratic function

TEKS 2A.4D Transform a quadratic function $f(x) = ax^2 + bx + c$ to the form $f(x) = a(x - h)^2 + k$ to identify the different attributes of $f(x)$.

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EXPLORE & SHARE

The table represents $A(x)$, the area of a square as a function of side length x units, where x is a positive real number.

Side Length (units)	x	1	2	3	4
Model	x				
Area (sq. units)	$A(x)$	1	4	9	16

A. Consider the function where the areas in the table are doubled. Write the equation of a function that represents this.

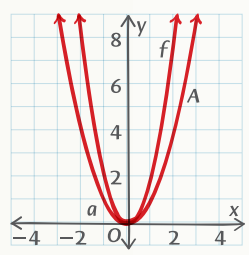
B. Communicate Math Ideas Graph the ordered pairs for both $A(x)$ and your new function. How would you describe the differences in the locations of these points?

C. Find the equation for a function whose x-values are the same as $A(x)$ but whose y-values are 2 units greater than each y-value in $A(x)$.

SAMPLE RESPONSES

A. $f(x) = 2(x^2)$

B.



The points that represent the new function are closer to the y-axis. The value of each y-value in the new function is twice that of the y-value in $A(x)$.

C. $g(x) = x^2 + 2$

Use with EXPLORE & SHARE

PROCESS STANDARDS CONNECTION

Communicate Math Ideas Suppose you started with a different function. Would altering $A(x)$ in the two ways you did have the same effect?

[Yes, each time you multiply the value of any function by 2, the *y*-value of each coordinate will double, and each time you add 2 to the value of any function, the *y*-value of each coordinate will increase by 2.]

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STEP 2 Understand & Apply

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Activity

INTRODUCE THE ESSENTIAL QUESTION

Establish Mathematics Goals to Focus Learning

Introduce students to the vertex form of a quadratic function. Remind them that they have identified transformations of linear functions. Explain that the vertex form of a quadratic function can help them identify transformations of a quadratic function.

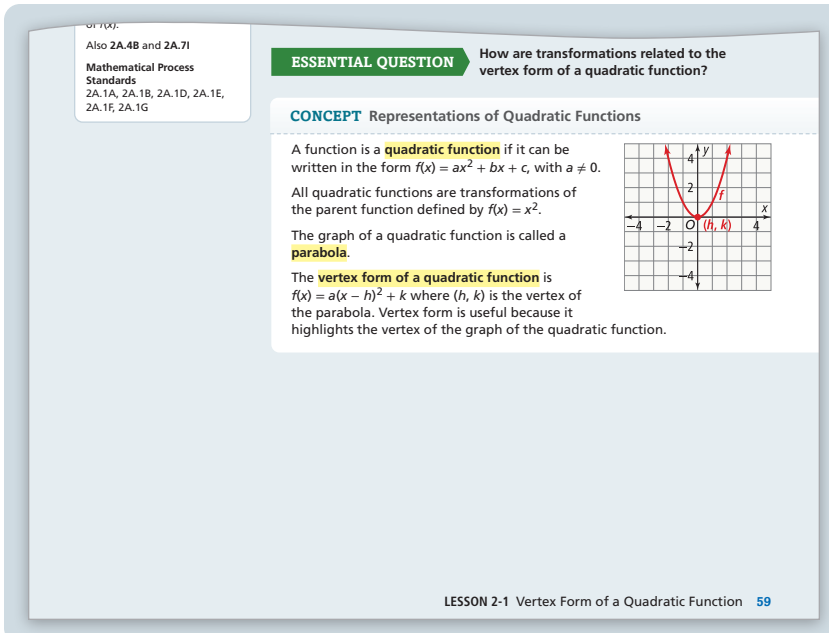
CONCEPT Representations of Quadratic Functions

Use and Connect Mathematical Representations

Q: What are the key features of the graph of the quadratic parent function $f(x) = x^2$?
[vertex at the origin; axis of symmetry is the y-axis; minimum: $y = 0$; domain: $(-\infty, \infty)$; range: $[0, \infty)$]

Q: What are the values a, h, and k for the quadratic parent function?
[$a = 1$, $h = 0$, $k = 0$]

Examples are available at SavvasRealize.com.


Also **2A.4B** and **2A.7I**

Mathematical Process Standards
2A.1A, 2A.1B, 2A.1D, 2A.1E, 2A.1F, 2A.1G

ESSENTIAL QUESTION How are transformations related to the vertex form of a quadratic function?

CONCEPT Representations of Quadratic Functions

A function is a **quadratic function** if it can be written in the form $f(x) = ax^2 + bx + c$, with $a \neq 0$.

All quadratic functions are transformations of the parent function defined by $f(x) = x^2$.

The graph of a quadratic function is called a **parabola**.

The **vertex form of a quadratic function** is $f(x) = a(x - h)^2 + k$ where (h, k) is the vertex of the parabola. Vertex form is useful because it highlights the vertex of the graph of the quadratic function.

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EXIT TICKET

Use this Exit Ticket anytime after Example 2.

ITEM	TEKS	DOK
1	2A.4B	2

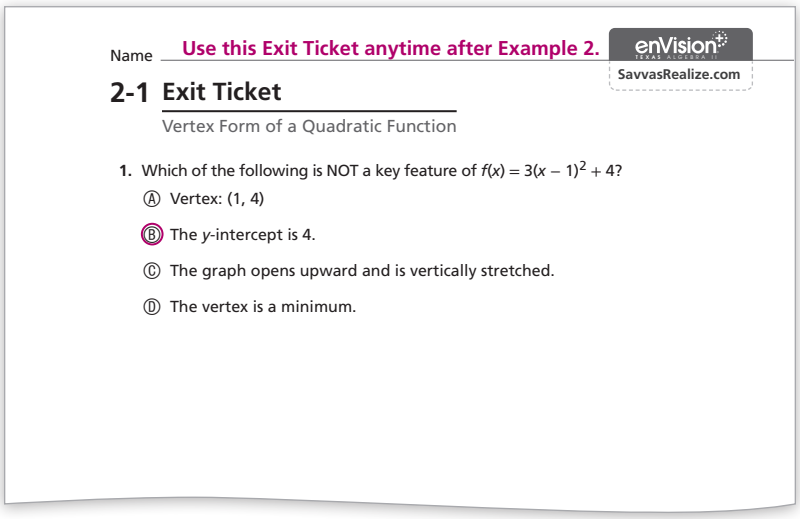
Use the Exit Ticket results to inform instructional decisions.

Instructional Implications

Students who choose an incorrect response do not understand how the function's equation is related to the transformed graph. As your instruction on vertex form continues, plan to address common errors and misconceptions.

- Remind students that the coordinates of the vertex of the parabola is $(-h, k)$.
- Explain that the graph of the function has a minimum value when $a > 0$ and it has a maximum value when $a < 0$.
- Encourage students to plot points to show the magnitude of a stretch or compression.
- Use Do You Understand item 4, if needed.

Formative Assessment The Exit Ticket is available at Savvas Realize® and in the Assessment Sourcebook.


Name Use this Exit Ticket anytime after Example 2.

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2-1 Exit Ticket
Vertex Form of a Quadratic Function

1. Which of the following is NOT a key feature of $f(x) = 3(x - 1)^2 + 4$?
   Ⓐ Vertex: (1, 4)
   Ⓑ The y-intercept is 4.
   Ⓒ The graph opens upward and is vertically stretched.
   Ⓓ The vertex is a minimum.

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Activity

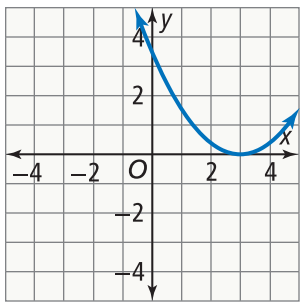
EXAMPLE 1 Transform a Quadratic Function

Use and Connect Mathematical Representations

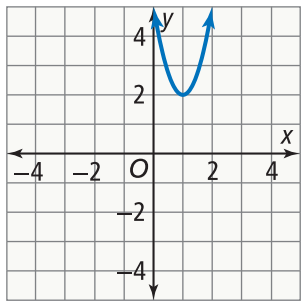
Q: How does the variable a affect the graph in relation to the parent function? [The value of a determines if the parabola is vertically stretched or vertically compressed and whether it opens upward or downward.]

Try It! Answers

1. a. The equation shows that the graph is vertically compressed, translated 3 units to the right, and will open upward.



b. The equation shows that the graph is vertically stretched, translated to the right 1 unit and up 2 units, and will open upward.



EXAMPLE 2 Determine Key Features of a Quadratic Function

Pose Purposeful Questions

Q: How do *a*, *h*, and *k* affect the domain and range of a quadratic function in vertex form?
[These values do not affect the domain of the function. The domain is all real numbers. These values affect the range. If *a* is positive, *k* is the minimum value of the range. If *a* is negative, *k* is the maximum value of the range.]

Try It! Answer

2. Vertex: (−4, −5); Axis of Symmetry: $x = -4$; Maximum: $y = -5$; Domain: $(-\infty, \infty)$; Range: $(-\infty, -5]$

Example 1 is Powered by Desmos.
Examples are available at SavvasRealize.com.

CONCEPTUAL UNDERSTANDING

EXAMPLE 1 Transform a Quadratic Function

How are transformations of the graph of $f(x) = x^2$ related to its parent function?

The vertex form of the quadratic function contains three values which transform the graph of the function $f(x) = x^2$.

$$f(x) = a(x - h)^2 + k$$

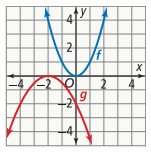
The value of *a* determines the direction the parabola opens and whether the graph is stretched or compressed.

The value of *h* determines the horizontal translation.

The value of *k* determines the vertical translation.

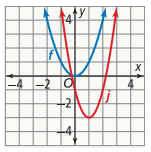
A. $g(x) = -\frac{1}{2}(x + 2)^2$

The equation shows that the graph is translated 2 units left, will open downward, and will be vertically compressed.



B. $j(x) = 2(x - 1)^2 - 3$

The equation shows that the graph is translated 1 unit right, 3 units down, and vertically stretched. It will open upward.



STUDY TIP
Recall that a vertical stretch makes the graph narrower and that a vertical compression makes the graph wider. To see the effects easily, use the same axes or units for all graphs.

When $a > 0$, the parabola opens upward. When $a < 0$, the parabola opens downward. When $|a| > 1$, the graph is stretched, and when $0 < |a| < 1$, the graph is compressed.

Try It! **1.** Describe the transformations of the parent function $f(x) = x^2$. Then graph the function.
a. $g(x) = 0.4(x - 3)^2$ **b.** $g(x) = 3(x - 1)^2 + 2$

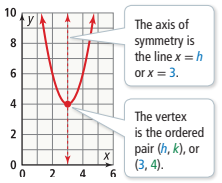
EXAMPLE 2 Determine Key Features of a Quadratic Function

What are the key features of the quadratic function $f(x) = 2(x - 3)^2 + 4$?

The graph shows $f(x) = 2(x - 3)^2 + 4$.

The 2 indicates that the graph opens upward and is vertically stretched.

The range is $\{y \mid y \geq 4\}$, so 4 is the minimum. There are no restrictions on the value of *x*, so the domain is all real numbers.



The axis of symmetry is the line $x = h$ or $x = 3$.

The vertex is the ordered pair (h, k), or $(3, 4)$.

Try It! **2.** Identify the vertex, axis of symmetry, minimum or maximum, domain, and range of the function $f(x) = -(x + 4)^2 - 5$.

60 TOPIC 2 Quadratic Functions and Equations

Use with **EXAMPLE 2**

PROCESS STANDARDS CONNECTION

Apply Math In what order should you apply the transformations shown in the Try It! for Example 2?

[reflection first, then vertical and horizontal translation]

Targeted ELPS Support WRITING 4E

(*Use with* **EXAMPLE 1**)

PRE-PRODUCTION Have students point to the blue curve in the first graph. Explain that this is the original, or parent graph. Then have them point to the red graph. Explain that this is the changed, or transformed graph. Repeat with the second graph.

BEGINNING Have students point to the blue curve in the first graph. Ask students to use their thumbs to point up or down to show which way the graph opens. After, have them say the word *upward*. Then repeat with the red graph. Have them say the word *downward*. Repeat with the second graph.

INTERMEDIATE Read the first call-out and ask students to point to the red *a* in the function. Ask students to gesture to show opening up or down. Repeat with the *h* having them show horizontal movement and *k* showing vertical movement.

HIGH INTERMEDIATE Have students copy the vertex form of the quadratic equation. Read each call-out and ask students to write their own labels describing *a*, *h*, and *k*.

ADVANCED Have pairs work together to write sentences relating the variables in the vertex form of the quadratic equation to the observations they can make in the graph. For example, "The graph moves 3 units down because *k* is −3".

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Activity

EXAMPLE 3 Write an Equation of a Parabola

Use and Connect Mathematical Representations

Q: Do you have to use both the vertex and the *y*-intercept to write the equation of a parabola?
[No, you do not have to use the *y*-intercept. You can use another point that lies on the parabola.]

Q: Why is the *y*-intercept a good point to use in this example?
[The *y*-intercept is easy to find from the graph. The calculations are simpler because the *x*-coordinate is 0.]

Try It! Answer

3. $y = \frac{1}{3}(x - 1)^2 - 4$

Elicit and Use Evidence of Student Thinking

Q: Which ordered pair would you use first to find the equation of the parabola?
[You use the coordinates of the vertex first and substitute for *h* and *k*.]

EXAMPLE 4 Write an Equation of a Parabola Given the Graph

Pose Purposeful Questions

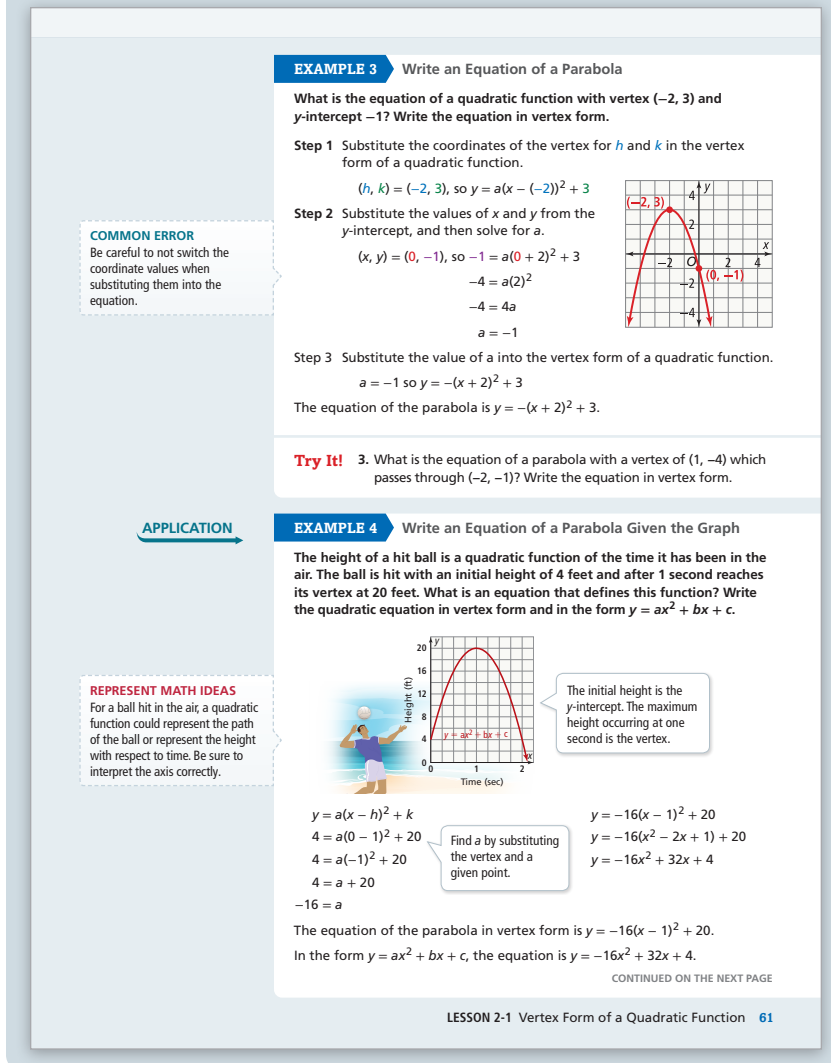
Q: What can you learn about the values of *a*, *b*, and *c* by looking at the graph?
[If the parabola opens upward, then *a* is positive. If it opens downward, then *a* is negative. The value of *c* is the *y*-coordinate of the *y*-intercept. You cannot determine the value of *b* by looking at the graph.]

Q: Why does the ball start with a height of 4?
[When the ball leaves the person's hand, it is 4 ft off the ground.]

Try It! Answer

4. $y = -\frac{3}{2}(x - 2)^2 + 10$; $y = -\frac{3}{2}x^2 + 6x + 4$

Examples are available at SavvasRealize.com.



EXAMPLE 3 Write an Equation of a Parabola

What is the equation of a quadratic function with vertex (–2, 3) and *y*-intercept –1? Write the equation in vertex form.

Step 1 Substitute the coordinates of the vertex for *h* and *k* in the vertex form of a quadratic function.

$(h, k) = (-2, 3)$, so $y = a(x - (-2))^2 + 3$

Step 2 Substitute the values of *x* and *y* from the *y*-intercept, and then solve for *a*.

$(x, y) = (0, -1)$, so $-1 = a(0 + 2)^2 + 3$
$-4 = a(2)^2$
$-4 = 4a$
$a = -1$

Step 3 Substitute the value of a into the vertex form of a quadratic function.

$a = -1$ so $y = -(x + 2)^2 + 3$

The equation of the parabola is $y = -(x + 2)^2 + 3$.

COMMON ERROR
Be careful to not switch the coordinate values when substituting them into the equation.

Try It! 3. What is the equation of a parabola with a vertex of (1, –4) which passes through (–2, –1)? Write the equation in vertex form.

APPLICATION

EXAMPLE 4 Write an Equation of a Parabola Given the Graph

The height of a hit ball is a quadratic function of the time it has been in the air. The ball is hit with an initial height of 4 feet and after 1 second reaches its vertex at 20 feet. What is an equation that defines this function? Write the quadratic equation in vertex form and in the form $y = ax^2 + bx + c$.

REPRESENT MATH IDEAS
For a ball hit in the air, a quadratic function could represent the path of the ball or represent the height with respect to time. Be sure to interpret the axis correctly.

The initial height is the *y*-intercept. The maximum height occurring at one second is the vertex.

$y = a(x - h)^2 + k$
$4 = a(0 - 1)^2 + 20$
$4 = a(-1)^2 + 20$
$4 = a + 20$
$-16 = a$

Find *a* by substituting the vertex and a given point.

$y = -16(x - 1)^2 + 20$
$y = -16(x^2 - 2x + 1) + 20$
$y = -16x^2 + 32x + 4$

The equation of the parabola in vertex form is $y = -16(x - 1)^2 + 20$.
In the form $y = ax^2 + bx + c$, the equation is $y = -16x^2 + 32x + 4$.

CONTINUED ON THE NEXT PAGE

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Extend Student Thinking

USE WITH EXAMPLE 2 Have students identify the key features of quadratic functions when the coordinates of the vertex are fractions.

1. $f(x) = -\frac{1}{2}\left(x - \frac{3}{2}\right)^2 + \frac{3}{4}$
2. $g(x) = \frac{2}{3}\left(x + \frac{1}{3}\right)^2 - \frac{1}{6}$

Q: Identify the vertex, axis of symmetry, minimum or maximum, domain, and range of the function.

[1. vertex: $\left(\frac{3}{2}, \frac{3}{4}\right)$
axis of symmetry: $x = \frac{3}{2}$
maximum at $y = \frac{3}{4}$
domain: $(-\infty, \infty)$
range: $\left(-\infty, \frac{3}{4}\right]$

2. vertex: $\left(-\frac{1}{3}, -\frac{1}{6}\right)$
axis of symmetry: $x = -\frac{1}{3}$
minimum at $y = -\frac{1}{6}$
domain: $(-\infty, \infty)$
range: $\left[-\frac{1}{6}, \infty\right)$]

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EXAMPLE 5 Write an Equation of a Transformed Function

Use and Connect Mathematical Representations

Q: What do you notice about the two graphs?
[The graphs open in the same direction. The graphs appear to have the same shape. The graph of g is shifted up and to the left.]

Q: What does this tell you about the value of a in the equations of f and g?
[Since g is a translation of f, the value of a does not change.]

Try It! Answers

5. a. $j(x) = (x - 2)^2 - 5$; $j(x) = x^2 - 4x - 1$

b. $j(x) = -(x - 0)^2 - 1$; $j(x) = -x^2 - 1$

Elicit and Use Evidence of Student Thinking

Q: How can you determine the value of a from the given information?
[In Part a, the graph of j is a translation of f. This means that $a = 1$. In Part b, the graph of j is a reflection of f in the x-axis. This means that $a = -1$.]

Common Error

Try It! 5a Some students may switch the coordinate values when writing the vertex form of the equation of the parabola. Emphasize that h is the x-coordinate of the vertex and k is the y-coordinate of the vertex.

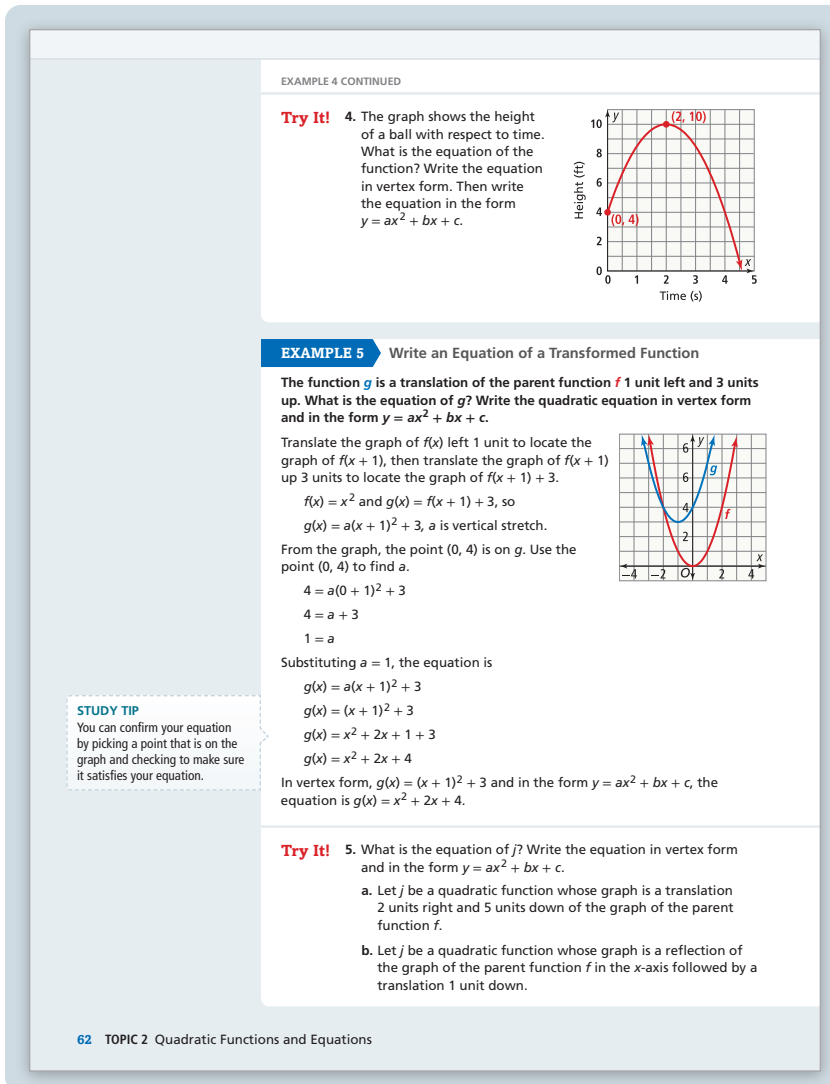
PROCESS STANDARDS CONNECTION

Use with EXAMPLE 5

Apply Math What information do you need to write the equation of a quadratic function in vertex form?

[the value of the vertex and at least one other point on the transformed function]

Examples are available at SavvasRealize.com.



EXAMPLE 4 CONTINUED

Try It! **4.** The graph shows the height of a ball with respect to time. What is the equation of the function? Write the equation in vertex form. Then write the equation in the form $y = ax^2 + bx + c$.

EXAMPLE 5 Write an Equation of a Transformed Function

The function g is a translation of the parent function f 1 unit left and 3 units up. What is the equation of g? Write the quadratic equation in vertex form and in the form $y = ax^2 + bx + c$.

Translate the graph of $f(x)$ left 1 unit to locate the graph of $f(x + 1)$, then translate the graph of $f(x + 1)$ up 3 units to locate the graph of $f(x + 1) + 3$.

$f(x) = x^2$ and $g(x) = f(x + 1) + 3$, so

$g(x) = a(x + 1)^2 + 3$, a is vertical stretch.

From the graph, the point (0, 4) is on g. Use the point (0, 4) to find a.

$4 = a(0 + 1)^2 + 3$

$4 = a + 3$

$1 = a$

Substituting $a = 1$, the equation is

$g(x) = a(x + 1)^2 + 3$

$g(x) = (x + 1)^2 + 3$

$g(x) = x^2 + 2x + 1 + 3$

$g(x) = x^2 + 2x + 4$

In vertex form, $g(x) = (x + 1)^2 + 3$ and in the form $y = ax^2 + bx + c$, the equation is $g(x) = x^2 + 2x + 4$.

STUDY TIP
You can confirm your equation by picking a point that is on the graph and checking to make sure it satisfies your equation.

Try It! **5.** What is the equation of j? Write the equation in vertex form and in the form $y = ax^2 + bx + c$.

a. Let j be a quadratic function whose graph is a translation 2 units right and 5 units down of the graph of the parent function f.

b. Let j be a quadratic function whose graph is a reflection of the graph of the parent function f in the x-axis followed by a translation 1 unit down.

62 TOPIC 2 Quadratic Functions and Equations

Support Student Understanding

USE WITH EXAMPLE 5 Some students may struggle with reflecting points across the x-axis.

Have students practice reflecting points across the x- and y-axes.

Q: Reflect (–3, 5) across the x-axis.
[(–3, –5)]

Q: Reflect (–4, –7) across the y-axis.
[(4, –7)]

Q: Reflect (4, 1) across the x-axis.
[(4, –1)]

Q: Reflect (2, –2) across the y-axis.
[(–2, –2)]

Language Routine: Compare and Connect

Organize students into groups of 4. Have each group count off so that each student has a number: 1, 2, 3, or 4. Display a graph of the function $f(x) = -2(x + 3)^2 - 1$ on a coordinate grid. Task each group with writing an equation of the graph in vertex form and in standard form. Give students time to ensure that everyone in the group can explain each part of the solution. Encourage students to take notes about their discussion and solution.

Call a random number from 1–4. The students with the number called are the reporters for their group. Reporters are allowed to use the notes that have already been created. Ask the reporters, one at a time, to explain the next step of the problem, to agree/disagree with the previous reporter, or to justify the reasoning of their group in some way. Correct answers are not revealed or agreed upon until every reporter shares.

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Presentation Practice

CONCEPT SUMMARY Vertex Form of a Quadratic Function

Q: What can you learn about the values of a, h, and k from the graph of g?
[The vertex is (–2, 3), so $h = -2$ and $k = 3$. Because the graph opens downward, the value of a is negative. And because g appears to be vertically compressed, $0 < |a| < 1$.]

Talk About Math Ideas

Listen for discussion that writing a quadratic function in vertex form reveals the vertex of the parabola (the maximum or minimum point) and allows for easier graphing and analysis of the function's behavior.

Q: Think about the situation you discussed during the Explore & Share. How are geometric squares related to algebraic squares?
[A square in geometry is a rectangle with equal-length sides. A square in algebra is the product of equal factors. The area of the geometric square is an algebraic square.]

Do You UNDERSTAND? | Do You KNOW HOW?

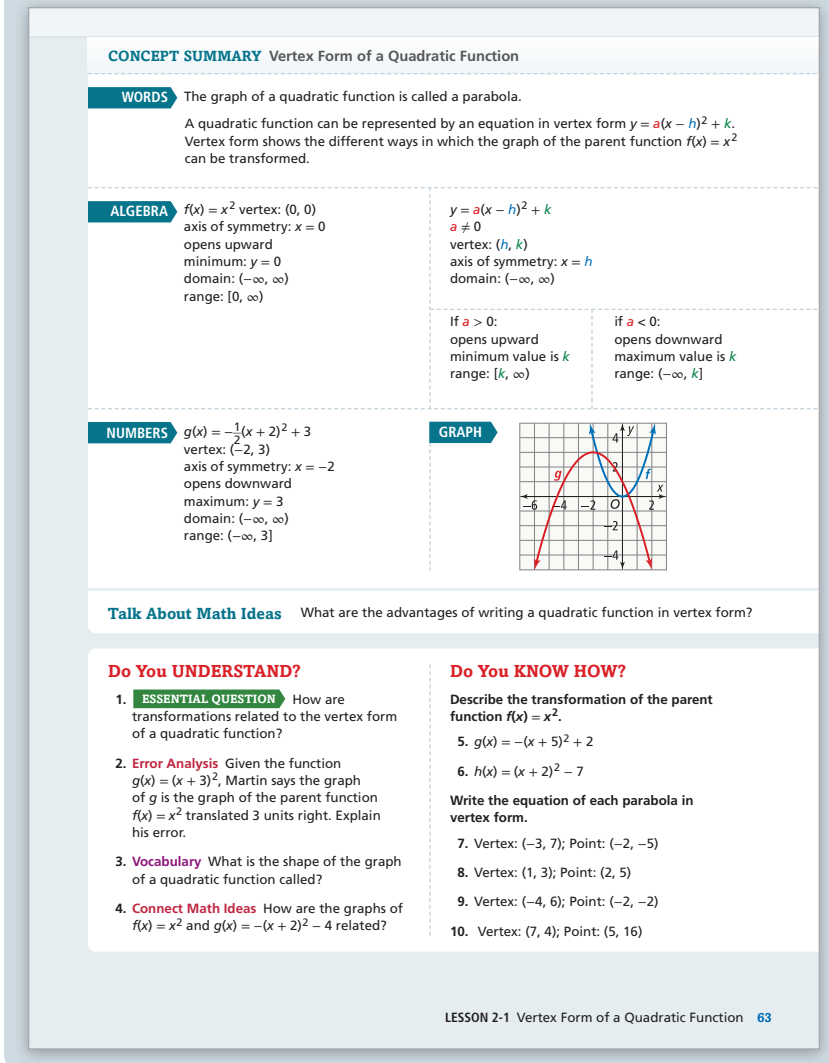
Common Error

Exercise 6 Some students may say that the function h is translated right 2 units instead of left 2 units. Emphasize that in the vertex form of a parabola, the value of h is subtracted from x, so $h = -2$, not 2.

Answers

1. Transformations move the points of a graph. The vertex shifts from the origin to (h, k) in the transformed function, where k is the vertical shift and h is the horizontal shift. These values are reflected in the vertex form, $y = a(x - h)^2 + k$. The graph may also be vertically stretched (if $|a| > 1$) or vertically compressed (if $|a| < 1$), and it may be reflected in the x-axis (if $a < 0$).
2. Martin should rewrite the function as $g(x) = (x - (-3))^2$. $h = -3$, and the parent function, $f(x) = x^2$, is translated 3 units left.
3. The shape of the graph of a quadratic function is called a parabola.
4. The graph of the function g is a reflection across the x-axis and a translation 2 units to the left and 4 units down of the graph of f.
5. The graph of the function g is a reflection across the x-axis and a translation 5 units to the left and 2 units up.
6. The graph of the function h is a translation 2 units to the left and 7 units down.
7. $y = -12(x + 3)^2 + 7$
8. $y = 2(x - 1)^2 + 3$
9. $y = -2(x + 4)^2 + 6$
10. $y = 3(x - 7)^2 + 4$
11. $g(x) = (x + 1)^2 + 3$

Concept Summary, Do You Understand, and **Do You Know How** are available at SavvasRealize.com.



CONCEPT SUMMARY Vertex Form of a Quadratic Function

WORDS The graph of a quadratic function is called a parabola.

A quadratic function can be represented by an equation in vertex form $y = a(x - h)^2 + k$. Vertex form shows the different ways in which the graph of the parent function $f(x) = x^2$ can be transformed.

ALGEBRA
$f(x) = x^2$ vertex: (0, 0)
axis of symmetry: $x = 0$
opens upward
minimum: $y = 0$
domain: $(-\infty, \infty)$
range: $[0, \infty)$

$y = a(x - h)^2 + k$
$a \neq 0$
vertex: (h, k)
axis of symmetry: $x = h$
domain: $(-\infty, \infty)$

If $a > 0$:
opens upward
minimum value is k
range: $[k, \infty)$

if $a < 0$:
opens downward
maximum value is k
range: $(-\infty, k]$

NUMBERS
$g(x) = -\frac{1}{2}(x + 2)^2 + 3$
vertex: (–2, 3)
axis of symmetry: $x = -2$
opens downward
maximum: $y = 3$
domain: $(-\infty, \infty)$
range: $(-\infty, 3]$

GRAPH

Talk About Math Ideas What are the advantages of writing a quadratic function in vertex form?

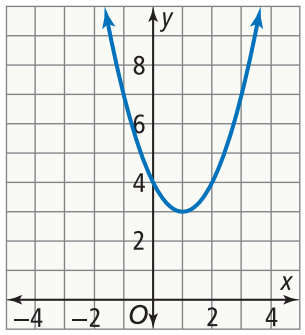
Do You UNDERSTAND?
1. ESSENTIAL QUESTION How are transformations related to the vertex form of a quadratic function?
2. Error Analysis Given the function $g(x) = (x + 3)^2$, Martin says the graph of g is the graph of the parent function $f(x) = x^2$ translated 3 units right. Explain his error.
3. Vocabulary What is the shape of the graph of a quadratic function called?
4. Connect Math Ideas How are the graphs of $f(x) = x^2$ and $g(x) = -(x + 2)^2 - 4$ related?

Do You KNOW HOW?
Describe the transformation of the parent function $f(x) = x^2$.
5. $g(x) = -(x + 5)^2 + 2$
6. $h(x) = (x + 2)^2 - 7$

Write the equation of each parabola in vertex form.
7. Vertex: (–3, 7); Point: (–2, –5)
8. Vertex: (1, 3); Point: (2, 5)
9. Vertex: (–4, 6); Point: (–2, –2)
10. Vertex: (7, 4); Point: (5, 16)

LESSON 2-1 Vertex Form of a Quadratic Function 63

12. No, the correct vertex is (–2, –4). The classmate should rewrite the function as $g(x) = -5(x - (-2))^2 - 4$ to see the signs for the values of the vertex (h, k).
13. $y = -2(x + 3)^2 + 4$
14. Since $a = -1$ and $-1 < 0$, the parabola opens downward.
15. a. The ball is 18 ft away from Amaya when it is at its maximum height. Use the x-coordinate of the vertex.
    b. Substitute 30 for the variable x in the equation. The ball's height is 9.12 ft.
16. The graph of the function is translated 1 unit right and 3 units up from the graph of $f(x) = x^2$.



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Practice

PRACTICE & PROBLEM SOLVING

Practice & Problem Solving, Adaptive Practice and **Spiral Review** are available at SavvasRealize.com.

Spaced and Interleaved Practice You may opt to assign the automatically scored lesson digital Practice and Problem Solving or prx: Spiral Review.

Assignment Guide

On-Level	Advanced
11, 12, 14, 16–38	11–15, 17, 19, 21–38

Engage Through Student Choice

Go online for an assignment strategy.

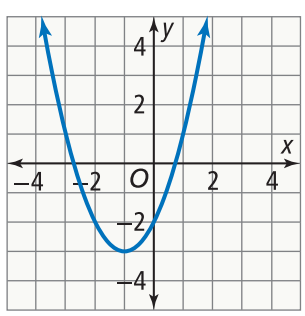
Item Analysis

Example	Item	DOK	TEKS Content	TEKS Process
1	16–21, 36	1	2A.4D	
2	22–25	1	2A.4D	
	12, 14, 32, 34, 38	2	2A.4B	2A.1G
	15, 18	3	2A.4D	2A.1A
3	26–28, 37	1	2A.4D	
	33	2	2A.4B	
	35	3	2A.4B	
4	29	1	2A.4B	
	13	2	2A.4D	
5	11, 30, 31	1	2A.4D	2A.1F

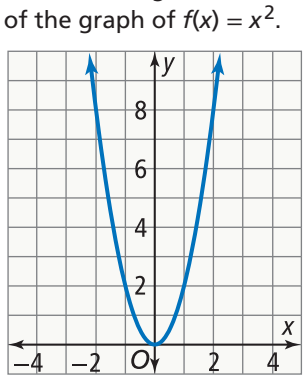
Answers

See answers for Problems 11–16 on previous page.

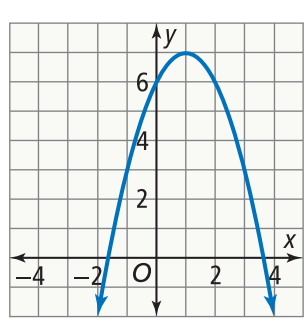
17. The graph of the function is translated 1 unit left and 3 units down from the graph of $f(x) = x^2$.



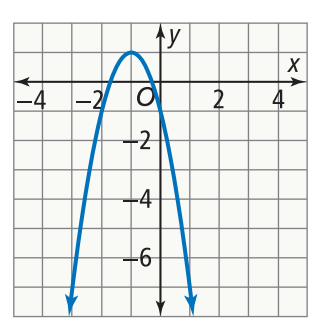
18. The function g is a vertical stretch of the graph of $f(x) = x^2$.



19. The graph of the function is a reflection in the x-axis, translated 1 unit right and 7 units up from the graph of $f(x) = x^2$.



20. The graph of the function is a reflection in the x-axis, translated 1 unit left and 1 unit up with a vertical stretch of the graph of $f(x) = x^2$.



See answers for Problems 21–31 on the next page.

PRACTICE & PROBLEM SOLVING

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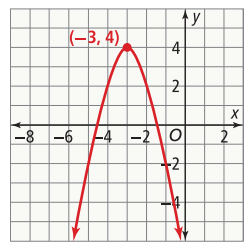
Lesson Help

UNDERSTAND

11. **Explain Math Ideas** The graph of the function $f(x) = x^2$ is translated 3 units up and 1 unit left. What is the resulting function $g(x)$?

12. **Error Analysis** A classmate said that the vertex of $g(x) = -5(x + 2)^2 - 4$ is (2, 4). Is your classmate correct? If not, what is the correct vertex?

13. **Higher Order Thinking** The graph below is a transformation of the graph of the parent function. Write the quadratic function to model the graph.



14. **Justify Math Arguments** Explain why the graph of the equation $g(x) = -(x + 1)^2 - 3$ would be a parabola opening downward.

15. **Use a Problem-Solving Model** Amaya is standing 30 ft from a volleyball net. The net is 8 ft high. Amaya serves the ball. The path of the ball is modeled by the equation $y = -0.02(x - 18)^2 + 12$, where x is the ball's horizontal distance in feet from Amaya's position and y is the distance in feet from the ground to the ball.

a. How far away horizontally is the ball from Amaya when it is at its maximum height? Explain.

b. Describe how you would find the ball's height when it crosses the net at $x = 30$.

PRACTICE

Describe the transformation of the parent function $f(x) = x^2$. Then graph the transformed function. SEE EXAMPLE 1

16. $j(x) = (x - 1)^2 + 3$ **17.** $y = (x + 1)^2 - 3$

18. $g(x) = 2x^2$ **19.** $h(x) = -(x - 1)^2 + 7$

20. $y = -2(x + 1)^2 + 1$ **21.** $g(x) = \frac{1}{2}(x - 2)^2 + 3$

Identify the vertex, axis of symmetry, maximum or minimum, domain, and range of each function. SEE EXAMPLE 2

22. $y = 2(x - 2)^2 + 5$ **23.** $f(x) = -(x - 1)^2 + 2$

24. $g(x) = -(x + 4)^2$ **25.** $y = \frac{1}{3}(x + 2)^2 - 1$

Write the equation of each parabola in vertex form. SEE EXAMPLE 3

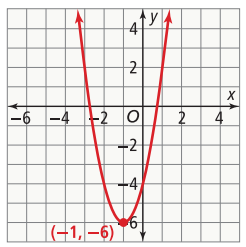
26. Vertex: (1, 2); Point: (2, −5)

27. Vertex: (3, 6); y-intercept: 2

28. Vertex: (0, 5); Point: (1, −2)

Write the equation of the function represented by the parabola in vertex form and in the form $y = ax^2 + bx + c$. SEE EXAMPLE 4

29.



Write the equation $g(x)$ in vertex form of a quadratic function given the transformations of the parent function $f(x) = x^2$. SEE EXAMPLE 5

30. Let $g(x)$ be the function whose graph is a translation 4 units left and 1 unit up of the graph of $f(x)$.

31. Let $g(x)$ be the function whose graph is a reflection in the x-axis and translated 3 units right of the graph of $f(x)$.

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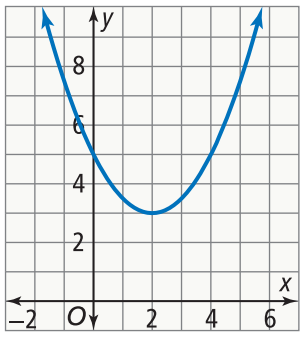
Practice

Answers

21. The graph of the function is a translation 2 units right and 3 units up with a vertical compression of the graph of $f(x) = x^2$.



22. Vertex: (2, 5); axis of symmetry: $x = 2$; minimum: $y = 5$; domain: $(-\infty, \infty)$; range: $[5, \infty)$

23. Vertex: (1, 2); axis of symmetry: $x = 1$; maximum: $y = 2$; domain: $(-\infty, \infty)$; range: $(-\infty, 2]$

24. Vertex: (–4, 0); axis of symmetry: $x = -4$; maximum: $y = 0$; domain: $(-\infty, \infty)$; range: $(-\infty, 0]$

25. Vertex: (–2, –1); axis of symmetry: $x = -2$; minimum: $y = -1$; domain: $(-\infty, \infty)$; range: $[-1, \infty)$

26. $y = -7(x - 1)^2 + 2$

27. $y = -\frac{4}{9}(x - 3)^2 + 6$

28. $y = -7x^2 + 5$

29. $y = 2(x + 1)^2 - 6$; $y = 2x^2 + 4x - 4$

30. $g(x) = (x + 4)^2 + 1$

31. $g(x) = -(x - 3)^2$

32. The maximum height for the shoes on the left is 16 in. The maximum height for the shoes on the right is 38 in. There is a 22-inch difference in the maximum heights.

33. Sample: (0.7 s, 2 m), (0.9 s, 0.24 m), (0.2 s, 2.98 m).

34. about 106 m

35. $y \approx -31.2(x - 2.5)^2 + 200$; Sample: Because we are interested in the vertical path, we need to ignore the horizontal distance. We could make a better model by recording more data points.

36. no; yes; no; no; no; yes

37. B

38. **Part A** The domain is $\{x \mid x > 0\}$. The value of x cannot be negative or 0 since it represents the price of a cookie.

Part B $f(0.40) = \$398.75$; $f(0.75) = \$355.00$

Part C The bakery should charge \$0.45 per cookie to make the maximum profit.

Part D The maximum profit the bakery can make is \$400.00.

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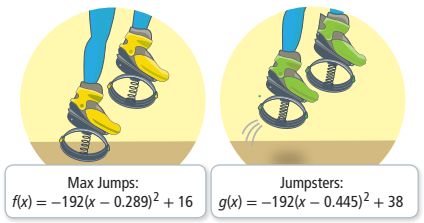
PRACTICE & PROBLEM SOLVING

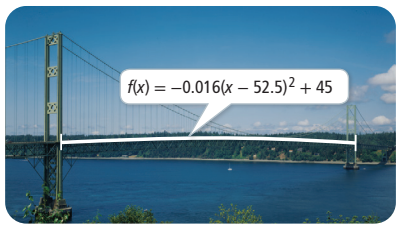
APPLY

32. **Communicate Reasoning** The height, in inches, that a person can jump while wearing jumping shoes is based on the time, x, in seconds. Kimani is testing two pairs to determine which ones she likes better. Compare the maximum heights on the two sets of shoes.



33. **Apply Math** A diver jumps from a 2 m board and 0.35 s later reaches his vertex at 3.2 m. Find his height at 3 other times.

34. **Communicate Reasoning** The curvature of the Tacoma Narrows Bridge in Washington is in the shape of a parabola.



In the given function, x represents the horizontal distance (in meters) from the arch's left end and $f(x)$ represents the vertical distance (in meters) from the base of the arch. What is the distance between the supports?

35. **Represent Math Ideas** An object is propelled from a height of 5 ft. After 2.5 s, the object reaches a maximum height of 200 ft, 150 horizontal feet away, and then it lands 5 s after it was propelled. Write the vertex form of the quadratic equation that models the object's vertical path. Explain why you used the values that you did and how you could make a better model.

ASSESSMENT PRACTICE

36. The graph of $g(x) = 3(x - 2)^2$ is a transformation of the graph of $f(x) = x^2$. Are the following transformations of f that map to g? Select *yes* or *no*. 2A.4D

	Yes	No
Translation left	☐	☐
Translation right	☐	☐
Translation up	☐	☐
Translation down	☐	☐
Vertical compression	☐	☐
Vertical stretch	☐	☐

37. **ACT** Which of the following functions represents a parabola with a vertex at (–3, 4) and that passes through the point (–1, –4)?

Ⓐ $f(x) = x^2 - 5$
Ⓑ $f(x) = -2(x + 3)^2 + 4$
Ⓒ $f(x) = 2(x + 1)^2 - 4$
Ⓓ $f(x) = 2(x - 3)^2 - 32$

38. **Performance Task** The Bluebird Bakery sells more alfajores when it lowers its prices, but this also changes profits. 2A.4B, 2A.4D, 2A.7I

The profit function for the alfajores is $f(x) = -500(x - 0.45)^2 + 400$. This function represents the profit earned when the price of an alfajor is x dollars. The bakery wants to maximize their profits.

Part A What is the domain of the function?

Part B Find the daily profits for selling alfajores for \$0.40 each and for \$0.75 each.

Part C What price should the bakery charge to maximize their profits from selling alfajores?

Part D What is the maximum profit?

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Assessment

QUICK CHECK

Use the Quick Check to assess students' understanding of the mathematics in the lesson.

Students can take the Quick Check online where it will be automatically scored and appropriate differentiated practice will be assigned based on student performance. Or, you can download a printable copy from **SavvasRealize.com**. The Quick Check is also available in the *Assessment Sourcebook*.

Item Analysis

Item	DOK	Skills Review and Practice	Standards
1	2	F60, F68	**2A.4B, 2A.4D**
2	2	F60	**2A.4B, 2A.4D**
3	2	F60	**2A.4B**
4	2	F16	**2A.4B, 2A.4D**
5	2	F16, F68	**2A.4B, 2A.4D**

DIFFERENTIATED RESOURCES

Points	Practice
0–3 points	• Reteach to Build Understanding • Mathematical Literacy and Vocabulary • Lesson Student Tutorial videos
4–5 points	• Additional Practice • Enrichment

Quick Check is available at SavvasRealize.com.

ASSESSMENT RESOURCES

Name _____ enVision TEXAS ALGEBRA 2 SavvasRealize.com

2-1 Quick Check

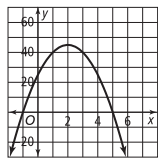
Vertex Form of a Quadratic Function

1. The function $g(x) = (x + 2)^2 + 4$ is a transformation of the parent function $f(x) = x^2$. Select all the statements that are true about the transformation.
   - ☐ A. Function g is the result of f being translated right 2 units and down 4 units.
   - ■ B. Function g is the result of f being translated left 2 units and up 4 units.
   - ■ C. The graph of function f opens upward.
   - ☐ D. The graph of function g opens downward.
   - ☐ E. The graph of function f is compressed by a factor of 2.

2. The function $h(x) = -\frac{1}{2}(x - 4)^2 + 3$ represents the height of a bird y over time x, as it flies past the school. What point on the graph represents the greatest height of the bird above the ground?
   - Ⓐ (4, 3)
   - Ⓑ (3, 4)
   - Ⓒ $\left(\frac{1}{2}, 4\right)$
   - Ⓓ $\left(3, \frac{1}{2}\right)$

3. What is the equation written in vertex form of a parabola with a vertex of (9, –1) that passes through (7, 7)?
   - Ⓐ $y = 2(x - 9)^2 - 1$
   - Ⓑ $y = 2(x + 1)^2 - 9$
   - Ⓒ $y = 2(x - 7)^2 + 7$
   - Ⓓ $y = 7(x - 9)^2 - 1$

4. Which of the following is an equation in the form $y = ax^2 + bx + c$ of the parabola shown in the graph?



   - Ⓐ $y = -5x^2 - 20x - 65$
   - Ⓑ $y = -5x^2 - 20x - 20$
   - Ⓒ $y = -x^2 - 27x - 90$
   - Ⓓ $y = -5x^2 + 20x + 25$

5. Function g is a transformation of the parent function $f(x) = x^2$. The graph of g is a translation right 3 units and down 5 units of the graph of f. What is the equation of function g written in the form $y = ax^2 + bx + c$?
   - Ⓐ $y = x^2 + 6x + 4$
   - Ⓑ $y = x^2 - 6x + 4$
   - Ⓒ $y = x^2 + 10x + 22$
   - Ⓓ $y = x^2 - 6x + 14$

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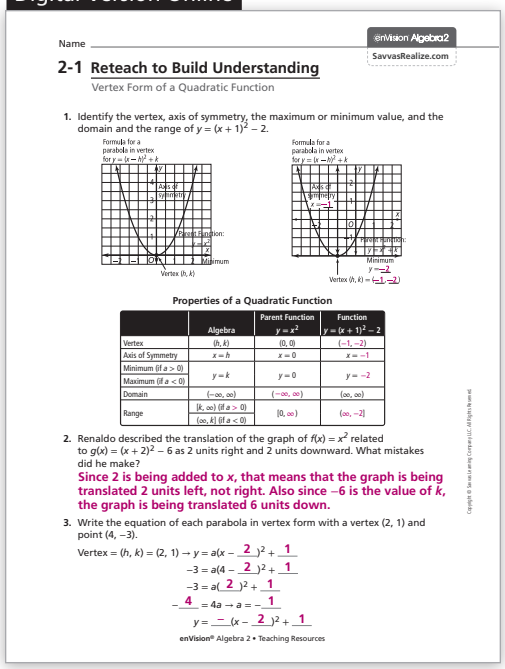
Practice

DIFFERENTIATED RESOURCES

Reteach to Build Understanding

Provides scaffolded reteaching for the key lesson concepts.

Digital Version Online



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2-1 Reteach to Build Understanding

Vertex Form of a Quadratic Function

1. Identify the vertex, axis of symmetry, the maximum or minimum value, and the domain and the range of $y = (x + 1)^2 - 2$.

Formula for a parabola in vertex form for $y = (x - h)^2 + k$: Axis of symmetry; Parent function: $y = x^2$; Vertex (h, k); Minimum

Formula for a parabola in vertex form for $y = (x - h)^2 + k$: Axis of symmetry $x = -1$; Parent function: $y = x^2$; Minimum $y = -2$; Vertex $(h, k) = (-1, -2)$

Properties of a Quadratic Function

	Algebra	Parent Function $y = x^2$	Function $y = (x + 1)^2 - 2$
Vertex	(h, k)	$(0, 0)$	$(-1, -2)$
Axis of Symmetry	$x = h$	$x = 0$	$x = -1$
Minimum (if $a > 0$)	$y = k$	$y = 0$	$y = -2$
Maximum (if $a < 0$)			
Domain	$(-\infty, \infty)$	$(-\infty, \infty)$	(∞, ∞)
Range	$[k, \infty)$ (if $a > 0$)	$[0, \infty)$	$(\infty, -2]$
	$(\infty, k]$ (if $a < 0$)		

2. Renaldo described the translation of the graph of $f(x) = x^2$ related to $g(x) = (x + 2)^2 - 6$ as 2 units right and 2 units downward. What mistakes did he make?
Since 2 is being added to x, that means that the graph is being translated 2 units left, not right. Also since −6 is the value of k, the graph is being translated 6 units down.

3. Write the equation of each parabola in vertex form with a vertex (2, 1) and point (4, −3).

Vertex = $(h, k) = (2, 1) \rightarrow y = a(x - \underline{2})^2 + \underline{1}$
$-3 = a(4 - \underline{2})^2 + \underline{1}$
$-3 = a(\underline{2})^2 + \underline{1}$
$-\underline{4} = 4a \rightarrow a = -\underline{1}$
$y = \underline{-}(x - \underline{2})^2 + \underline{1}$

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Additional Practice

Provides extra practice for each lesson.

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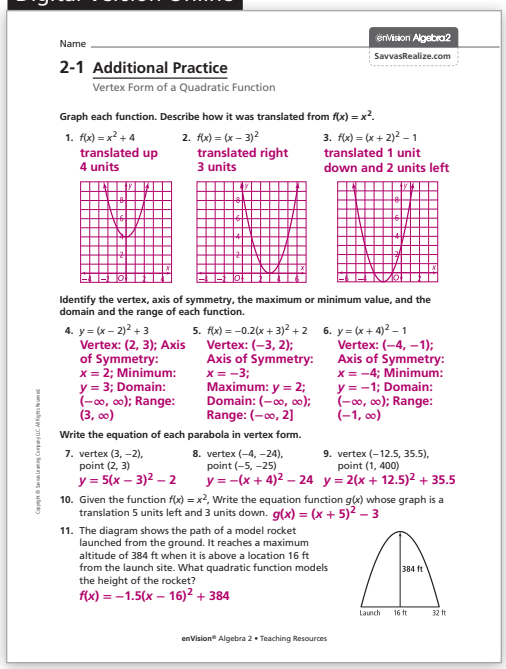

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2-1 Additional Practice

Vertex Form of a Quadratic Function

Graph each function. Describe how it was translated from $f(x) = x^2$.

1. $f(x) = x^2 + 4$
translated up 4 units

2. $f(x) = (x - 3)^2$
translated right 3 units

3. $f(x) = (x + 2)^2 - 1$
translated 1 unit down and 2 units left

Identify the vertex, axis of symmetry, the maximum or minimum value, and the domain and the range of each function.

4. $y = (x - 2)^2 + 3$
Vertex: (2, 3); Axis of Symmetry: $x = 2$; Minimum: $y = 3$; Domain: $(-\infty, \infty)$; Range: $(3, \infty)$

5. $f(x) = -0.2(x + 3)^2 + 2$
Vertex: (−3, 2); Axis of Symmetry: $x = -3$; Maximum: $y = 2$; Domain: $(-\infty, \infty)$; Range: $(-\infty, 2]$

6. $y = (x + 4)^2 - 1$
Vertex: (−4, −1); Axis of Symmetry: $x = -4$; Minimum: $y = -1$; Domain: $(-\infty, \infty)$; Range: $(-1, \infty)$

Write the equation of each parabola in vertex form.

7. vertex (3, −2), point (2, 3)
$y = 5(x - 3)^2 - 2$

8. vertex (−4, −24), point (−5, −25)
$y = -(x + 4)^2 - 24$

9. vertex (−12.5, 35.5), point (1, 400)
$y = 2(x + 12.5)^2 + 35.5$

10. Given the function $f(x) = x^2$, Write the equation function $g(x)$ whose graph is a translation 5 units left and 3 units down. **$g(x) = (x + 5)^2 - 3$**

11. The diagram shows the path of a model rocket launched from the ground. It reaches a maximum altitude of 384 ft when it is above a location 16 ft from the launch site. What quadratic function models the height of the rocket?
$f(x) = -1.5(x - 16)^2 + 384$

384 ft; Launch; 16 ft; 32 ft

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Enrichment

Presents engaging problems and activities that extend the lesson concepts.

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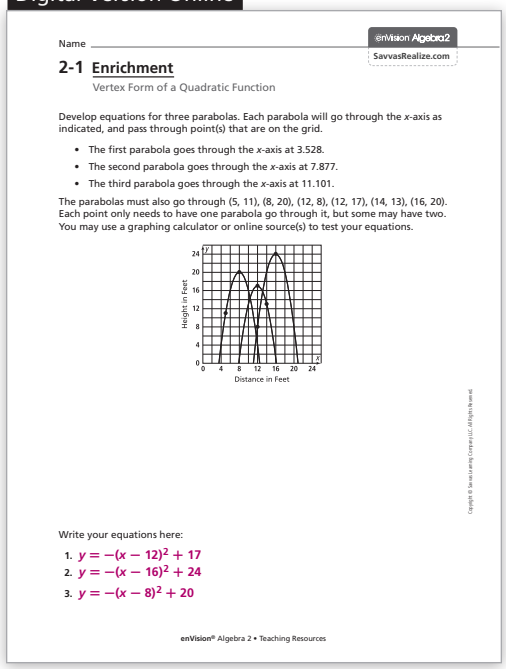

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2-1 Enrichment

Vertex Form of a Quadratic Function

Develop equations for three parabolas. Each parabola will go through the x-axis as indicated, and pass through point(s) that are on the grid.

- The first parabola goes through the x-axis at 3.528.
- The second parabola goes through the x-axis at 7.877.
- The third parabola goes through the x-axis at 11.101.

The parabolas must also go through (5, 11), (8, 20), (12, 8), (12, 17), (14, 13), (16, 20). Each point only needs to have one parabola go through it, but some may have two. You may use a graphing calculator or online source(s) to test your equations.

Height in Feet; Distance in Feet

Write your equations here:

1. **$y = -(x - 12)^2 + 17$**
2. **$y = -(x - 16)^2 + 24$**
3. **$y = -(x - 8)^2 + 20$**

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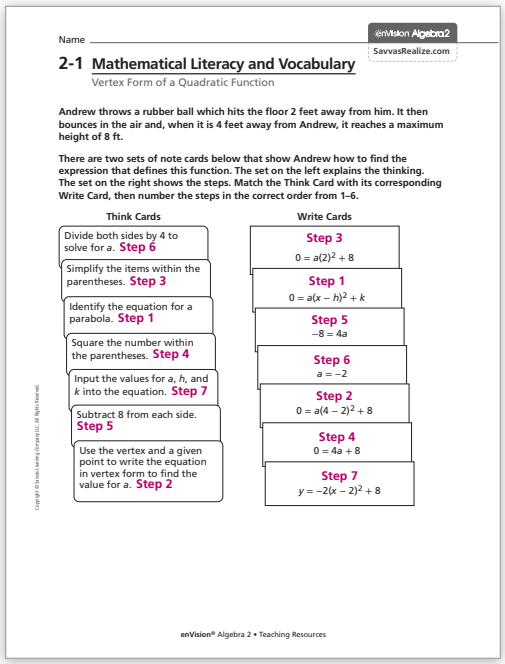
Mathematical Literacy and Vocabulary

Helps **emergent bilingual** students develop and reinforce understanding of key terms and concepts.



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2-1 Mathematical Literacy and Vocabulary

Vertex Form of a Quadratic Function

Andrew throws a rubber ball which hits the floor 2 feet away from him. It then bounces in the air and, when it is 4 feet away from Andrew, it reaches a maximum height of 8 ft.

There are two sets of note cards below that show Andrew how to find the expression that defines this function. The set on the left explains the thinking. The set on the right shows the steps. Match the Think Card with its corresponding Write Card, then number the steps in the correct order from 1–6.

Think Cards

- Divide both sides by 4 to solve for a. **Step 6**
- Simplify the items within the parentheses. **Step 3**
- Identify the equation for a parabola. **Step 1**
- Square the number within the parentheses. **Step 4**
- Input the values for a, h, and k into the equation. **Step 7**
- Subtract 8 from each side. **Step 5**
- Use the vertex and a given point to write the equation in vertex form to find the value for a. **Step 2**

Write Cards

- **Step 3** $0 = a(2)^2 + 8$
- **Step 1** $0 = a(x - h)^2 + k$
- **Step 5** $-8 = 4a$
- **Step 6** $a = -2$
- **Step 2** $0 = a(4 - 2)^2 + 8$
- **Step 4** $0 = 4a + 8$
- **Step 7** $y = -2(x - 2)^2 + 8$

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Expand the Learning

You may choose to provide time in class to discuss innovations in the mathematics field by individuals highlighted in the **Amazing Contributions** Teacher Resource Original found on SavvasRealize.com. You may wish to provide the Teacher Resource Original found on SavvasRealize.com. Consider allowing time for students to share responses to the THINK question.

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TOPIC 2

Math Modeling in 3 Acts: Swift Kick

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Video / Audio

Lesson Overview

KEY CONCEPTS

Mathematics Objectives

- ✔ Use mathematical modeling to represent a problem situation and to propose a solution.
- ✔ Test and verify the appropriateness of their math models.
- ✔ Explain why the results from their mathematical models might not align exactly with the problem situation.

Language Objective

- ✔ Use precise mathematical language with peers to discuss strategies and identify problem-solving methods.

Essential Understanding

Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly.

COHERENCE

Earlier in this topic, students:

- Solved systems of equations.
- Explored different forms of quadratic functions.

In this lesson, students:

- Develop a mathematical model to represent and propose a solution to a problem situation involving quadratic functions.

Later in this topic, students will:

- Refine their mathematical modeling skills by solving quadratic equations and linear-quadratic systems.

BALANCE

This mathematical modeling lesson focuses on application of both math content and math processes.

- Students draw on their understanding of concepts related to quadratic functions to develop a representative model.
- Students apply their mathematical model to test and validate its applicability to similar problem situations.

MATH MODELING IN 3 ACTS

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Mathematical Process Standards
2A.1A Apply mathematics to problems arising in everyday life, society, and the workplace. Also **2A.1B, 2A.1E,** and **2A.1G**

TEKS
2A.4A, 2A.8C

Swift Kick

Whether you call it soccer, football, or fútbol, it's the most popular sport in the world by far. Even if you don't play soccer, you probably know several people who do.

There are many ways to kick a soccer ball: you can use any part of either foot. If you want the ball to end up in the goal, you also need to try different amounts of spin and power. You'll see one person's effort in the Math Modeling in 3-Acts lesson.

ACT 1

ACT 1 Identify the Problem

1. What is the first question that comes to mind after watching the video?
2. Write down the main question you will answer about what you saw in the video.
3. Make an initial conjecture that answers this main question.
4. Explain how you arrived at your conjecture.
5. What information will be useful to know to answer the main question? How can you get it? How will you use that information?

BUILD G.R.I.T. Resilience
It is okay to struggle. If one approach to finding a model is not working, try a different approach.

ACT 2 Develop a Model

6. Use the math that you have learned in this Topic to refine your conjecture.

ACT 3 Interpret the Results

7. Did your refined conjecture match the actual answer exactly? If not, what might explain the difference?

88 TOPIC 2 Math Modeling in 3 Acts

Student Companion

Students can do their work for the task on their *Student Companion* pages, which you can print from **SavvasRealize.com.**

Texas Essential Knowledge and Skills (TEKS)

Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

2A.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

2A.1A apply mathematics to problems arising in everyday life, Number Flexibility to 100 for All Four Operations

2A.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

Also **2A.1E, 2A.1F, 2A.1G**

Quadratic and square root functions, equations, and inequalities. The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

2A.4A write the quadratic function given three specified points in the plane Data. The student applies mathematical processes to analyze data, select appropriate models, write corresponding functions, and make predictions.

2A.8C predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models.

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Video / Audio

Swift Kick

In this mathematical modeling task, students analyze partial views of soccer shots to determine whether the attempts are successful. They need to determine whether the ball's path will pass through the goal. To do so, they apply concepts they study in Topics 1 and 2.

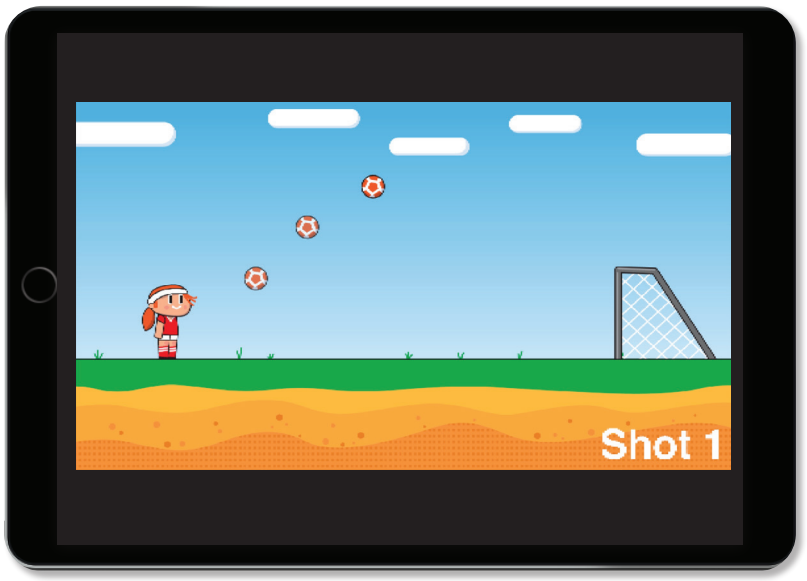
ACT 1 The Hook

Play the video. The video shows a video game character shooting soccer balls in profile. Each shot traces part of a parabola. The video does not show the result of any of the attempts.

After the question brainstorming, have students consider the Main Question. Remind students to write down their questions and conjectures.

MAIN QUESTION

Which of the shots go in?



ACT 2 Modeling With Math

Think about the task. Ask students to speculate how they can determine which shots go in. Then have them think about what information they need.

Reveal the information. Use the video, which includes a coordinate grid and sample points for each shot, to give students specific information about the paths. Then, you can use the graphing calculator to explore quadratic functions that model each shot.

What's the connection? Give students time to struggle as they think about how to connect their ideas to what they learned in this topic about quadratic functions. How could they use the information to write a function for each shot? How can they use those functions to solve the problem?

Before showing the answers, find out where there is disagreement. Allow students to argue their cases and update their conjectures.

INTERESTING MOMENTS WITH STUDENTS

Students may assume that every curve includes the vertex. Shots 3 and 4 stop at the vertex, but the other shots stop before the vertex. Students may incorrectly think they can use symmetry for all curves to determine the paths of the shots. Invite a student who uses this method to explain it during Act 3. This may lead to a conversation about how Shot 4 is a counterexample to this conjecture.

Necessary Information

Horizontal distance: 36 ft
Height of goal: 8 ft

Coordinates

- Shot 1: (5, 7) and (15, 15)
- Shot 2: (6, 2.8) and (15, 6.25)
- Shot 3: (10, 13.75) and (16, 16)
- Shot 4: (11, 8.25) and (22, 11)
- Shot 5: (3, 2.5) and (12, 8)

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Video / Audio

ACT 3 The Solution

Play the video. The final video shows the entirety of the five shots. The outcome of each shot is revealed, including an overhead angle. After each shot, ask students for a show of hands of whose initial conjecture was accurate.

MAIN QUESTION ANSWER

Shot 1: The ball goes in the top-right corner.

Shot 2: The ball misses high.

Shot 3: The ball goes in on one bounce.

Shot 4: The ball is the right height but misses left.

Shot 5: The ball misses. It hits the crossbar.

Do the "post-game" analysis. Help students understand why a shot's result may not match what the parabola predicts. Students should focus on how the paths are in 2-D and do not show the third dimension. They may not have considered that Shot 3 could bounce in or that Shot 4 has an accurate path from one perspective but not from another. Look for creative ideas students offer to improve the method for predicting whether a shot goes in.

Now that the solution has been revealed and discussed, ask students to analyze and evaluate the efficiency of the approach they chose to solve the problem.

ONE POSSIBLE SOLUTION

For each shot, substitute the two given points (x, y) and the origin into the standard form of a parabola. Start with Shot 1.

Use (0, 0).	Use (5, 7).	Use (15, 15).
$y = ax^2 + bx + c$	$y = ax^2 + bx + c$	$y = ax^2 + bx + c$
$0 = a(0)^2 + b(0) + c$	$7 = a(5)^2 + b(5) + c$	$15 = a(15)^2 + b(15) + c$
$0 = c$	$7 = 25a + 5b + c$	$15 = 225a + 15b + c$

Since $c = 0$, the result is a system of two linear equations.

$25a + 5b = 7$

$225a + 15b = 15$

The solution of the system is $a = -0.04$, $b = 1.6$.

Now, substitute a, b, and c into standard form.

$y = -0.04x^2 + 1.6x$

Finally, substitute 36 for x.

$y = -0.04(36)^2 + 1.6(36)$

$y = 5.76$

The height of the ball when it reaches the goal is 5.76 ft.

Since $5.76 < 8$, the shot will go in the goal.

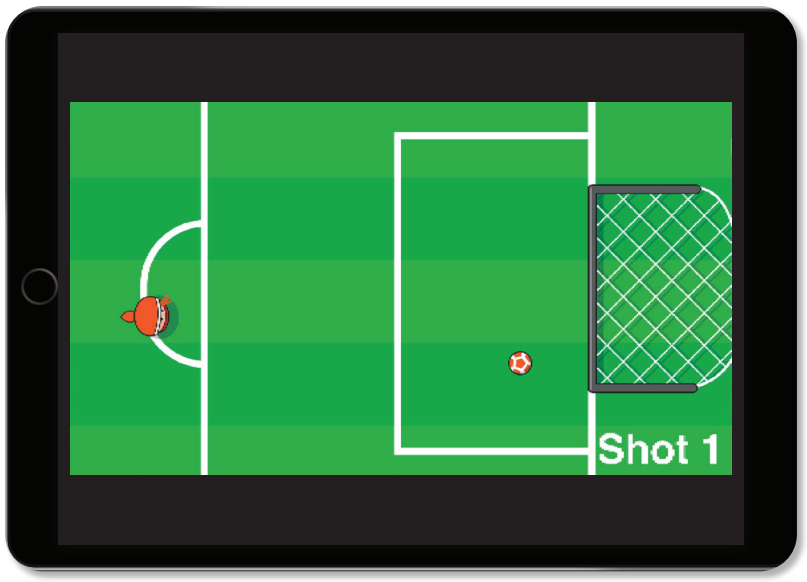
Repeat for Shots 2–5.

Shot 2: $y = -0.00\overline{5}x^2 + 0.5x$; when $x = 36$, $y = 10.8$; shot misses

Shot 3: $y = -0.0625x^2 + 2x$; when $x = 36$, $y = -9$; shot goes in

Shot 4: $y = -0.02\overline{27}x^2 + x$; when $x = 36$, $y \approx 6.55$; shot goes in

Shot 5: $y = -0.0\overline{185}x^2 + 0.\overline{8}x$; when $x = 36$, $y = 8$; shot misses



INTERESTING MOMENTS WITH STUDENTS

Shots 3, 4, and 5 are designed to start a discussion. Students can discuss how the ground, the third dimension, and the boundaries of the goal influence the result. For example, students may think that a shot that passes through (36, 8) will barely go in when it actually hits the crossbar of the goal.

CREATE A PROBLEM

As students finish, have them construct their own problem involving a projectile. Challenge students to require finding the solutions to a linear-quadratic solution or a quadratic inequality. Then have them trade with a partner to solve.

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